The Government Expenditure Multiplier in a Real Model

Juan Herreño

April 11, 2018

1 Introduction

The focus of our attention will be a partial derivative $\frac{\partial Y_t}{\partial G_t}$. This partial derivative will change across models. The reason is that the government multiplier is not a structural parameter, it is instead an elasticity that will depend on the specifics of the model. The goal of Woodford (2011) is to understand what are reasonable (under the lenses of standard theory) values of the multiplier for the two big families of standard RA models we have (the RBC and the NK model), in order to check if a well-estimated aggregate government multiplier in the data can help us to distinguish across models.

The answer is that government expenditure multipliers that are estimated using aggregate data in times of normal monetary policy are consistent both with the RBC and the NK models, therefore this multiplier (again, normal times of monetary policy using aggregate data) are not a good statistic to distinguish if we live in a world closer to the RBC model or to the NK model. Today we will focus on the real model.

1.1 RBC model

1.1.1 Household

Households maximize

$$\sum_{t=0}^{\infty} \beta^t \left[u(C_t) - v(H_t)\right] \tag{1}$$

Subject to a standard budget constraint Intra-period FOC implies that:

$$\frac{v'(H_t)}{u'(C_t)} = \frac{W_t}{P_t} \tag{2}$$

1.1.2 Firms

Firms produce with a production function

$$Y_t = f(H_t) \tag{3}$$

Profit maximization implies that:

$$f'(H_t) = \frac{W_t}{P_t} \tag{4}$$

1.1.3 Market Clearing

In the aggregate

$$Y_t = C_t + G_t \tag{5}$$

1.1.4 Multiplier

Equalizing the labor supply equation (2) to the labor demand equation (4 we get:

$$\frac{v'(H_t)}{u'(C_t)} = f'(H_t) \tag{6}$$

We can now use the production function to replace H away, and the aggregate demand equation to replace C away.

$$u'(Y_t - G_t) = \tilde{v}'(Y_t) \tag{7}$$

Realize that Y is an endogenous variable. In particular $Y = Y(G)$. Now totally differentiate the above equation with respect to G.

$$u''(Y-G)\frac{dY}{dG} - u''(Y-G) = \tilde{v}''(Y)\frac{dY}{dG} \tag{8}$$

Now divide both sides by u', realize that $u' = \tilde{v}'$:

$$-\frac{u''}{u'} = \frac{dY}{dG}\left(\frac{\tilde{v}''}{\tilde{v}'} - \frac{u''}{u'}\right) \tag{9}$$

And define the elasticities as $\eta_u = -\bar{Y}\frac{u''}{u'}$ and $\eta_v = \bar{Y}\frac{u''}{u'}$

$$\frac{dY}{dG} = \frac{\eta_u}{\eta_u + \eta_v} \tag{10}$$

2 Experiments

The way of thinking this is the following. Imagine you had an amount of coconuts, and the government takes away from you G of those coconuts. You have the chance to produce more coconuts in response to the change in G. Consuming less coconuts hurts your utility function (you like coconuts), but produce more coconuts requires labor, and you value leisure. The value of the multiplier then depends how much government expenditures crowd-out consumption. If equilibrium consumption is constant for different levels of government expenditures, that is if households supply as much labor as necessary in order to keep C constant, then the multiplier is equal to 1. If households let their consumption to fall for the full G amount, then the multiplier is 0.

What determines how much crowding out we will have in the model? 1) preferences as we saw in the previous simple model, and 2) the available technology to smooth consumption intertemporally.

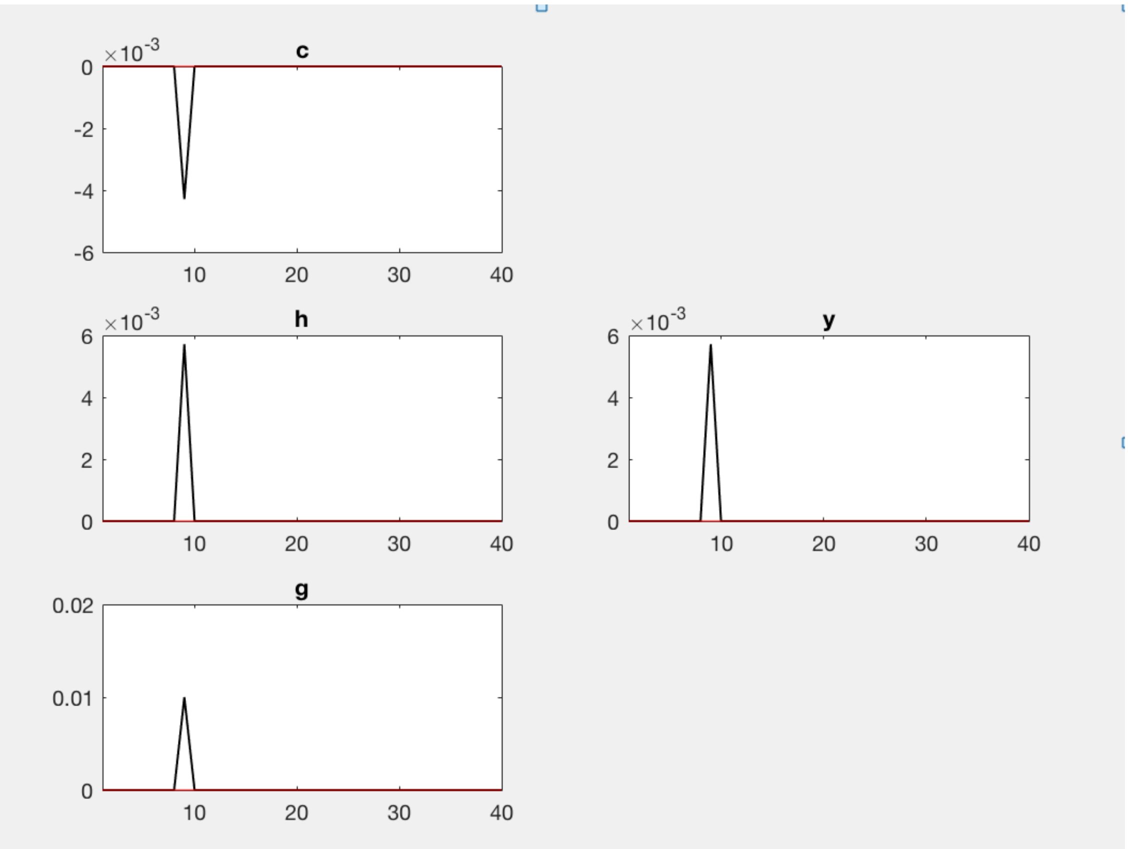

Figure 1: News Shock without capital. Increase happens in period 8 but it is announced in period 0

2.1 News Shock - No Capital

This one is easy after you think about it. You cannot move coconuts from one period to the next. You know tomorrow you will have a hard time, but you cannot do anything about it.

The only important thing is to predict the behavior of the real interest rate.

We know that always the real interest rate (maybe from a notional bond that does not trade in equilibrium!) is given by:

$$\frac{u'(C_t)}{\beta u'(C_{t+1})} = R_t$$

We know that tomorrow C_{t+1} will be lower, therefore $u'(C_{t+1})$ will be higher, therefore the real interest rate needs to go down. What's the intuition? Imagine that you could trade the bond. R_t going down is the price that would make you to choose a path of consumption equal to the one you choose in the absence of the bond.

2.2 Temporary increase in G - Capital

To do this exercise we will need the computer. I will assume the following functional forms:

$$u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}$$
$$v(H_t) = \frac{H_t^{1+\nu}}{1+\nu}$$

$$f(K_t, H_t) = K_t^{\alpha} H_t^{1-\alpha}$$

for simplicity I'm going to write the first order conditions after plugging in the equilibrium prices of labor and capital, so the FOCs will depend on the marginal products of capital and labor.

$$H_t^{\nu} = C_t^{-\gamma}(1-\alpha)K_t^{\alpha} H_t^{-\alpha}$$
$$Y_t = C_t + I_t + G_t$$
$$Y_t = K_t^{\alpha} H_t^{1-\alpha}$$
$$K_{t+1} = K_t(1-\delta) + I_t$$
$$C_t^{-\gamma} = \beta \mathbb{E}_t C_{t+1}^{-\gamma}\left(1 - \delta + \alpha K_t^{\alpha-1} H_t^{1-\alpha}\right)$$

The question we face is what happens with the endogenous variables Y, C, I, H after a purely transitory shock G.

I am going to assume that the steady state value of G is zero.

In order to start by what we know, let's start with a value $\alpha \to 0$. That model is close to the model without capital. The results are in figure 1.

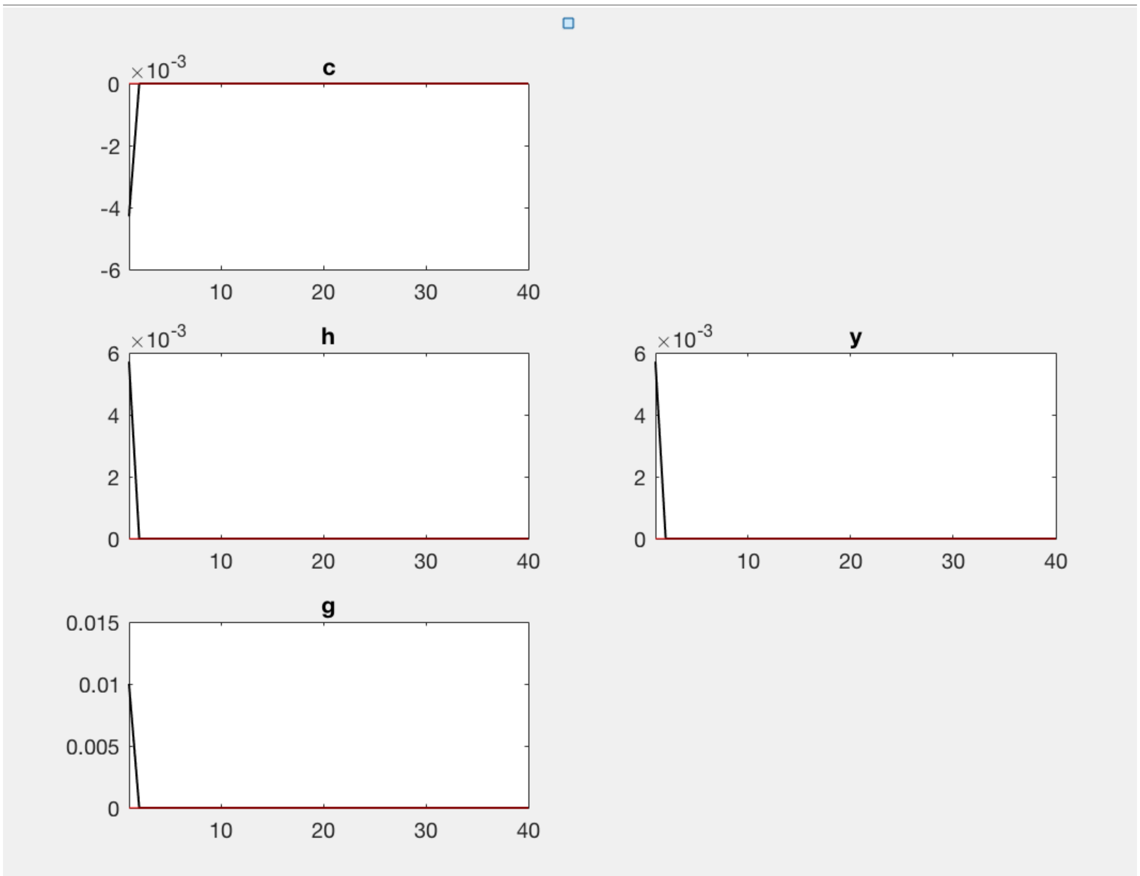

Figure 2: Response of endogenous variables to an increase in g with no capital

Now we are going to take $\alpha = 0.3$.

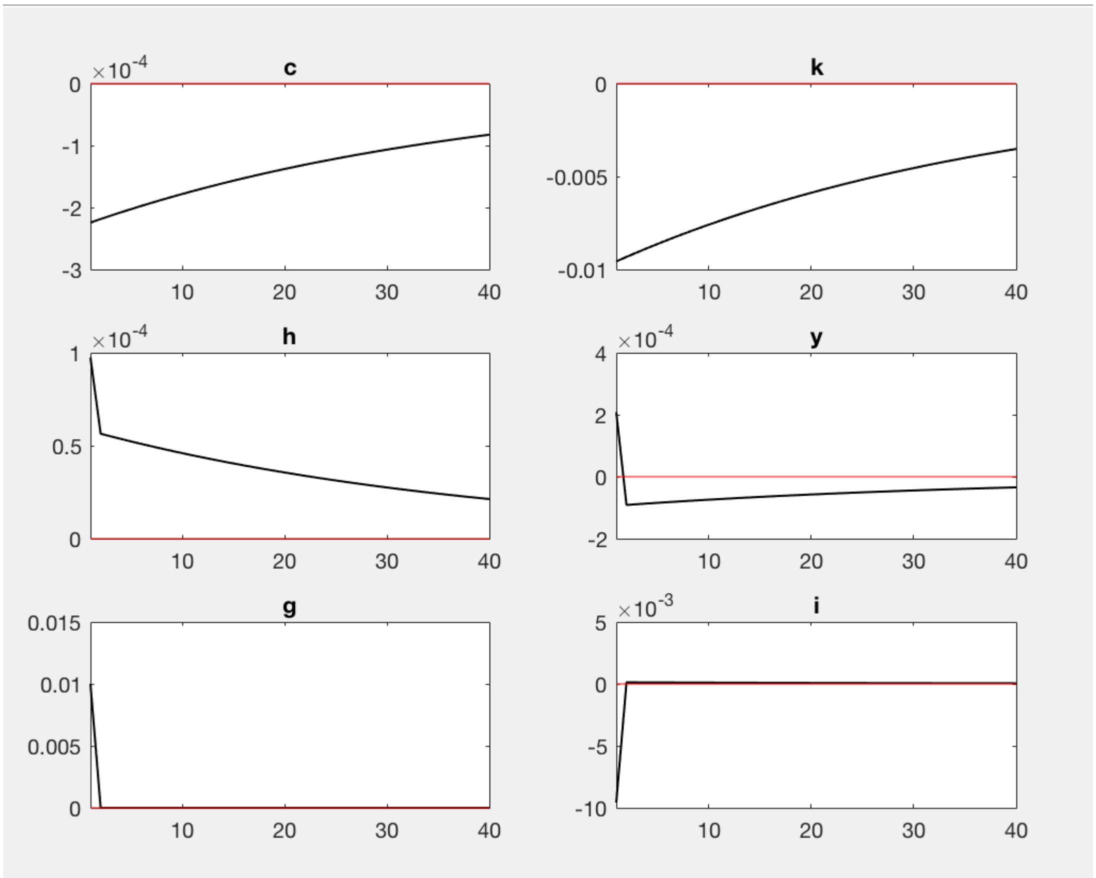

Figure 3: Response of endogenous variables to a temporary increase in g with capital

Here the figure k shows k_{t+1}, not k_t. There are two main message. First, the multiplier is significantly smaller on impact. Why? Because the agent has an additional margin of adjustment. On the demand side $Y = C + I + G$, so C does not have to fall as much to accomodate an increase in G, since now I can fall. On the supply side K is fixed at time 0, so the adjustment has to come through H. The household can smooth consumption by lowering investment, reducing the capital stock in the future in such a way that the marginal utility of consumption is smoothed. Because C does not fall by much, then H does not have to increase by much, therefore Y does not increase as much.

Second, note the effects on output in subsequent periods. Output falls now, why? because capital will be below trend and the hours do not raise as much as to counteract it.

2.3 News about increase in G - Capital

Here the key is that you can do something about the shock. In particular you know that at a distant date, you are going to lose some coconuts but now you can transfer coconuts from the present to the future. What are you going to do? You will invest more to build your capital stock. Because your stock of capital is higher, then you want to work more, as the marginal product of labor also increases. You will consume a bit less in order to build your capital stock faster (if you are working more, then you will consume less by the wealth effect). When the time comes that you lose some coconuts you will draw down on investment as you don't need high capital stock in the future, which makes that the amount of labor goes down, and the variables then start to recover.

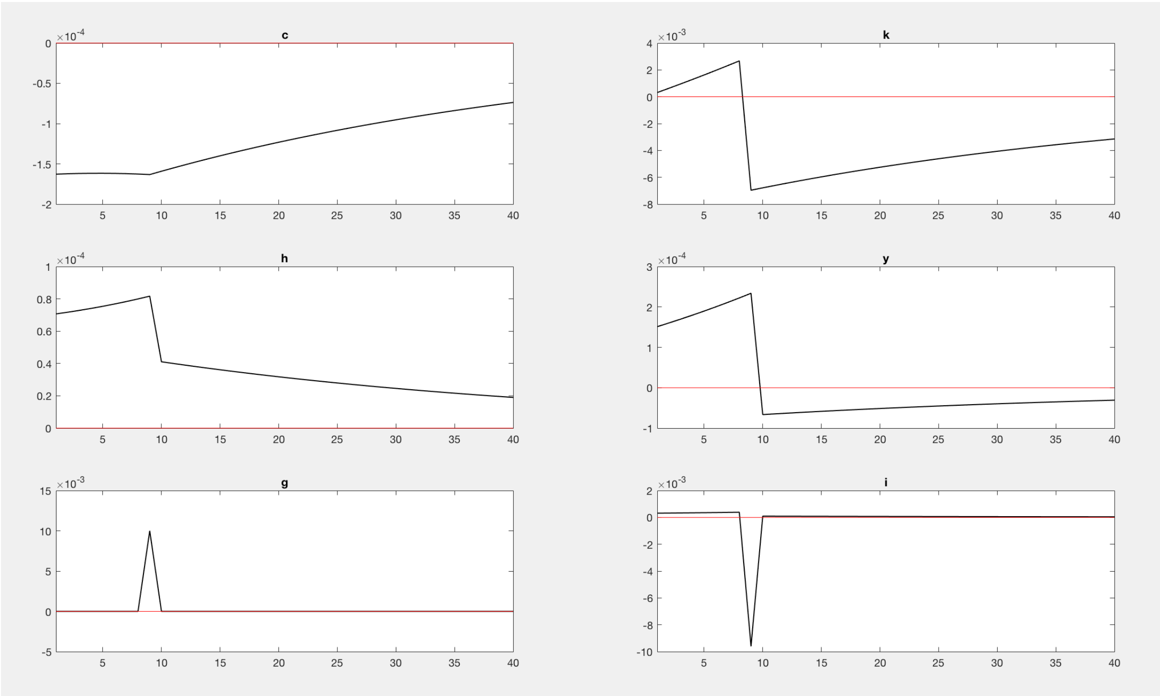

Figure 4: Response of endogenous variables to a news shock - increase in g in period 8 announced in period 0 - with capital

Then we check what is the effect of the same experiment in a world with high depreciation. Intuitively we want to put limits to the ability of the household to transfer resources across states.

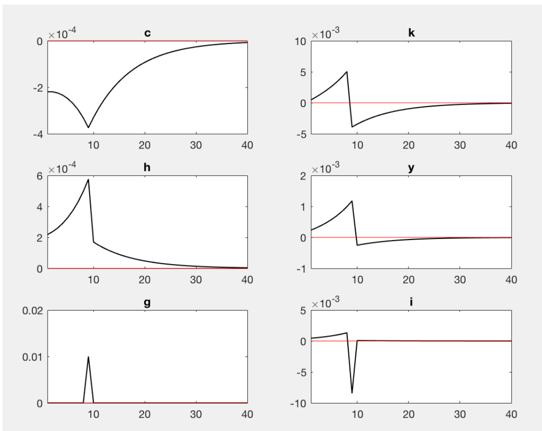

Figure 5: Response of endogenous variables to a news shock - increase in g in period 8 announced in period 0 - with capital - high depreciation