Recitation 4
Strategic Complementarities

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In any model, adjustment to any shock can happen via prices or quantities. What we call monetary non-neutrality is that after a change in a nominal variable (money supply, nominal interest rates) output reacts. We have studied a mechanism by which this can happen, which we called price rigidity. We said price rigidity makes the aggregate price level to move sluggishly, and since prices do not fully react, then quantities do.

But why is the price level not adjusting in the economy with price rigidity? Imagine two scenarios. In scenario one, every firm gets to adjust its price, and they decide to raise their prices by 1% in response to a shock. In scenario two, only one firm gets to adjust its price and decides to adjust it by 10% after the shock. If all the prices have the same weight in the price index, then inflation will be the same in the two alternative scenarios. It has to be the case, than in the sitcky price economy firms that are adjusting their price, are not adjusting enough as to compensate for those who are not able to adjust.

Today, we are going to discuss the concept of strategic complementarities, which are characteristics of the model economies that make price rigidity strong.

Pricing decisions are strategic complements if an increase in the prices charged for other goods **increases** the price that it is optimal to charge for one's own good (Woodford, 2003). More to the point, the question we want to ask is, once we control for nominal aggregate demand, how my price depends on the price of others.

We care about strategic complementarities, because they make the effect of price rigidity stronger. If there a lot of strategic complementarities, then firms that can adjust their price would not do it by much, since they would like to stay close to the aggregate price index, which is dominated by the majority of firms that are not adjusting. Therefore, strategic complementarities + price rigidity makes the aggregate price level to move sluggishly, which implies that the effects of monetary policy shocks are long-lived.

What we are after is a relationship of the type:

$$p_{it} = \alpha + \beta m_t + \delta x_{it} + \omega p_t$$

Where lower case variables denote variables in logs, $M = PY$ is nominal aggregate demand, x are firm-level cost shifters, and p is the price level. Pricing decisions are strategic complements if $\omega > 0$, or strategic substitutes if $\omega < 0$. Note that we require prices to be close to the price index for reasons that go beyond being close to nominal aggregate demand (which depends on p of course).

0.1 Simple model without strategic complementarities

The utility function is

$$U(C_t, N_t) = \log C_t - N_t$$

The standard household problem, which I won't write for brevity is characterized by three equations.

$$W_t = P_t C_t$$
$$C_{it} = C_t \left(\frac{P_{it}}{P_t}\right)^{-\sigma}$$
$$\frac{1}{C_t} = \beta(1+i_t)\mathbb{E}_t\left[\frac{1}{C_{t+1}}\Pi_{t+1}^{-1}\right]$$

The firm problem is given by:

$$\max_{P_{it}} \pi_{it} = P_{it}Y_{it} - W_t N_{it}$$
$$\text{subject to}$$
$$C_{it} = C_t \left(\frac{P_{it}}{P_t}\right)^{-\sigma}$$
$$Y_{it} = C_{it}$$
$$Y_{it} = A_{it}N_{it}$$

Plugging all the constraints together into the objective function:

$$\max_{P_{it}} \pi_{it} = P_{it}Y_t \left(\frac{P_{it}}{P_t}\right)^{-\sigma} - \frac{W_t}{A_{it}}Y_t \left(\frac{P_{it}}{P_t}\right)^{-\sigma}$$

With first order condition:

$$P_{it}^* = \frac{\sigma}{\sigma-1}\frac{W_t}{A_{it}}$$

And using the household FOC $W_t = P_t C_t$ together with market clearing condition $C_t = Y_t$ and definition $M_t = P_t Y_t$, we get:

$$P_{it}^* = \frac{\sigma}{\sigma-1}\frac{M_t}{A_{it}}$$

Or in logs:

$$p_{it}^* = \log\frac{\sigma}{\sigma-1} + m_t - a_{it} \tag{1}$$

That is, once we control for nominal aggregate demand, prices do not depend on aggregate prices. This model exhibits zero strategic complementarities. We can deduce that if we introduce price rigidity in this model, the absence of strategic complementarities will make it hard for monetary policy to have long-lasting effects.

0.2 Decreasing Returns to Scale

We now make a small change, named that the production function is now:

$$Y_{it} = A_{it}L_{it}^{\alpha}$$

You can see that the conditions that come from the household are not altered, named:

$$W_t = P_t C_t$$
$$C_{it} = C_t \left(\frac{P_{it}}{P_t}\right)^{-\sigma}$$

$$\frac{1}{C_t} = \beta(1+i_t)\mathbb{E}_t\left[\frac{1}{C_{t+1}}\Pi_{t+1}^{-1}\right]$$

But now the profit function of the firm is a bit different:

$$\max_{P_{it}} \pi_{it} = P_{it}Y_t\left(\frac{P_{it}}{P_t}\right)^{-\sigma} - \frac{W_t}{A_{it}^{1/\alpha}}\left(Y_t\left(\frac{P_{it}}{P_t}\right)^{-\sigma}\right)^{1/\alpha}$$

We take first order conditions and find that p_{it}^* depend on aggregate prices.