Recitation 3
The non-linear Calvo Model

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1 Introduction

So far we have derived a set of first order conditions for the Calvo model, and after linearize them, we arrived to a system of three equations. The Euler equation, the NK Phillips Curve, and a monetary policy rule. How much we lose by linearizing? How does the steady state of this model looks like?

2 The Calvo Model

I'll start by writing some optimality conditions for this model. You should know by now how to derive them, if you don't for a particular one, make sure to review the material.

I'm going to assume the following utility function:

$$U(C_t, N_t) = \frac{C_t^{1-\gamma} - 1}{1-\gamma} - \frac{N_t^{1+\psi}}{\psi} \tag{1}$$

$$N_t^{\psi} = \frac{W_t Y_t^{-\gamma}}{P_t} \,:\, \text{Labor supply} \tag{2}$$

$$Y_t^{-\gamma} = \beta(1+i_t)\mathbb{E}_t \left[Y_{t+1}^{1-\gamma} \Pi_{t+1}^{-1} \right] \,:\, \text{Euler Equation} \tag{3}$$

$$N_t = \frac{Y_t D_t}{A_t} \,:\, \text{Aggregate Production Function} \tag{4}$$

$$D_t = \int_0^1 \left(\frac{P_{it}}{P_t} \right)^{-\sigma} di \,:\, \text{Price Dispersion} \tag{5}$$

$$P_t^* = \frac{X_{1,t}}{X_{2,t}} \,:\, \text{Target Price} \tag{6}$$

$$X_{1,t} = \frac{\sigma}{\sigma - 1} Y_t^{1-\gamma} P_t^{\sigma} \frac{W_t}{P_t A_t} + \beta\theta \mathbb{E}_t \left[X_{1,t+1} \right] \tag{7}$$

$$X_{2,t} = Y_t^{1-\gamma} P_t^{\sigma-1} + \beta\theta \mathbb{E}_t \left[X_{2,t+1} \right] \tag{8}$$

$$Y_t^n = \left(\frac{\sigma}{\sigma - 1} A_t^{\psi+1}\right)^{\frac{1}{\psi+\gamma}} \text{ : Natural Output} \tag{9}$$

$$\frac{W_t}{P_t} = Y_t^{\psi+\gamma} D_t^{\psi} A_t^{-\psi} \text{ : Output} \tag{10}$$

$$\tilde{Y}_t = \frac{Y_t}{Y_t^n} \text{ : Output Gap} \tag{11}$$

$$P_t^{1-\sigma} = (1-\theta)(P_t^*)^{1-\sigma} + \theta(P_{t-1})^{1-\sigma} \tag{12}$$

$$(1+i_t) = \beta^{-1}\bar{\Pi}\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi} e^{v_t} \text{ : Monetary Policy Rule} \tag{13}$$

Now we are going to transform this system in terms of inflation rates, as opposed to price indexes. Start by defining:

$$K_{1,t} \equiv \frac{X_{1,t}}{P_t^{\sigma}} = \frac{\sigma}{\sigma-1} Y_t^{1-\gamma} \frac{W_t}{P_t A_t} + \beta\theta \mathbb{E}_t \left[K_{1,t+1}\Pi_{t+1}^{\sigma}\right] \tag{14}$$

$$K_{1,t} \equiv \frac{X_{1,t}}{P_t^{\sigma}} = Y_t^{1-\gamma} \tilde{Y}_t^{\gamma+\psi} D_t^{\psi} + \beta\theta \mathbb{E}_t \left[K_{1,t+1}\Pi_{t+1}^{\sigma}\right] \tag{15}$$

Where I divided (10) by (9) and replaced it by $\frac{W_t \sigma}{(\sigma-1)P_t A_t}$

$$K_{2,t} \equiv \frac{X_{1,t}}{P_t^{\sigma-1}} = Y_t^{1-\gamma} + \beta\theta \mathbb{E}_t \left[K_{2,t+1}\Pi_{t+1}^{\sigma-1}\right] \tag{16}$$

Now using (16), (14) and (6) and dividing over P_{t-1} on both sides we get:

$$\Pi_t^* = \Pi_t \frac{K_{1,t}}{K_{2,t}} \tag{17}$$

Now take equation (5):

$$D_t = \int_0^{1-\theta} \left(\frac{P_t^*}{P_t}\right)^{-\sigma} dj + \int_{1-\theta}^{1} \left(\frac{P_{i,t-1}}{P_t}\right)^{-\sigma} dj$$

$$D_t = (1-\theta)\left(\Pi_t^*\right)^{-\sigma} \Pi_t^{\sigma} + \theta D_{t-1} \Pi_t^{\sigma} \tag{18}$$

Now use equation (12):

$$\Pi_t^{1-\sigma} = \theta + (1-\theta)(\Pi_t^*)^{1-\sigma} \tag{19}$$

So the system is given by:

$$\Pi_t^{1-\sigma} = \theta + (1-\theta)(\Pi_t^*)^{1-\sigma}$$
$$D_t = (1-\theta)\left(\Pi_t^*\right)^{-\sigma}\Pi_t^\sigma + \theta D_{t-1}\Pi_t^\sigma$$
$$\Pi_t^* = \Pi_t \frac{K_{1,t}}{K_{2,t}}$$
$$K_{1,t} \equiv \frac{X_{1,t}}{P_t^\sigma} = Y_t^{1-\gamma}\tilde{Y}_t^{\gamma+\psi}D_t^\psi + \beta\theta\mathbb{E}_t\left[K_{1,t+1}\Pi_{t+1}^\sigma\right]$$
$$K_{2,t} \equiv \frac{X_{1,t}}{P_t^{\sigma-1}} = Y_t^{1-\gamma} + \beta\theta\mathbb{E}_t\left[K_{2,t+1}\Pi_{t+1}^{\sigma-1}\right]$$
$$\tilde{Y}_t = \frac{Y_t}{Y_t^n}$$
$$Y_t^n = \left(\frac{\sigma}{\sigma-1}A_t^{\psi+1}\right)^{\frac{1}{\psi+\gamma}}$$
$$Y_t^{-\gamma} = \beta(1+i_t)\mathbb{E}_t\left[Y_{t+1}^{1-\gamma}\Pi_{t+1}^{-1}\right]$$
$$(1+i_t) = \beta^{-1}\bar{\Pi}\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi}e^{v_t}$$
$$\log A_t = \rho\log A_{t-1} + u_t$$

Let's look how the steady state solution looks like. I'm going to do it for an inflation target $\bar{\Pi} = 1$. You'll do the more involved case in the homework.

From the Euler Equation:

$$\Pi = \beta(1+i)$$

and from the monetary policy rule:

$$(1+i) = \beta^{-1}\Pi^{\phi_\Pi}$$
$$\Pi = \Pi^{\phi_\Pi}$$
$$\Pi = 1$$

From the definition of inflation as a function of target inflation:

$$1 = \theta + (1-\theta)(\Pi^*)^{1-\sigma}$$
$$1-\theta = (1-\theta)(\Pi^*)^{1-\sigma}$$
$$\Pi^* = 1$$

From the evolution of dispersion:

$$D = (1-\theta)\left(\Pi^*\right)^{-\sigma}\Pi^\sigma + \theta D\Pi^\sigma$$
$$D = \left(\Pi^*\right)^{-\sigma}$$
$$D = 1$$

Plugging in the efinition of K_1 and K_2

$$K_1 = \frac{Y^{1-\gamma}\tilde{Y}^{\gamma+\psi}D^\psi}{1-\beta\theta}$$
$$K_2 = \frac{Y^{1-\gamma}}{1-\beta\theta}$$

Therefore

$$\Pi^* = \Pi\frac{K_1}{K_2}$$

$$1 = \tilde{Y}^{\gamma+\psi} D^{\psi}$$

$$\tilde{Y} = 1$$

With the equilibrium conditions and the steady state, we can ask the computer to solve the model.