# Recitation 6
# Review Session

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## 1 Story Telling

We started the semester with a model similar to that you used in the Fall semester. And we made a small change by introducing nominal variables. The difference between nominal and real variables is that while the latter are denominated in units of goods, the former are denominated in currency.

We found that by only making this change little happened. The real variables were completely determined without any regard to nominal variables, a result we called the Classical Dichotomy; and the fact that nominal variables do not enter in the utility function meant that any monetary policy was equivalent from the perspective of the household. That is, in our standard model, money did not matter.

We then moved to considerate what happened if we included price rigidity, the intuition is that in any model the adjustment after shocks can happen through prices and quantities. If prices are rigid, could a particular shock create fluctuations in quantities? Before including price rigidity we realized we needed to give firms the power to set prices. We introduced the simplest way to do this, by introducing monopolistic competition in a Dixit-Stiglitz fashion. By doing this we realized that we were introducing a source of inefficiency in the economy. Because firms want to maximize profits they under-produce and under-employ. The economy with monopolistic competition was less efficient than the previous economy we consider. We realized that fixing this problem was easy under the assumption that lump-sum taxes are feasible policy measures. By including a payroll subsidy, firms would find cheaper to hire more. The optimal subsidy is the one that would take us back to the first best. Once we understood this we moved to introduce price rigidity, and we spent the semester using a particular assumption about the price-setting of firms, known as the Calvo model. We used the Calvo model because of its tractability, it let us to introduce staggered-price adjustment, capture the fact that prices stay fixed for sustained periods of time, but at the same time let us to characterize the solution analytically.

We showed that the steady state of the Calvo model with zero target inflation was the same as that of the flexible price equilibrium, that is the only difference arise from deviations with respect to steady state. We showed how this is not true when target inflation is different than zero. The intuition is that in a non-zero inflation steady state, the price level has some drift, and firms in their intention of tracking aggregate prices need to make price adjustments. Since not all of them can adjust at the same time, this introduces price dispersion in steady state. This result let us to understand the relationship between price dispersion and the output gap. Having price dispersion is inefficient, because goods that have the same marginal cost and enter in the same way in the utility of the household, are priced differently. This means that price dispersion that arises from price rigidity is seen in the aggregate as an inefficiency in the allocation.

We then showed that the Calvo model collapses to a system of three equations. an intertemporal IS equation (otherwise known as the Euler equation), a NK Phillips Curve (or aggregate supply relation), and a monetary policy rule (usually a Taylor Rule). We learned under which conditions of the monetary policy rule the bounded equilibrium was unique, and we called this the Taylor Principle. We learned that monetary policy only has an effect on output when the interventions are unexpected, because otherwise the inflation expectations shift, and output does not change.

We characterized the welfare loss of the economy under in a zero target inflation environment. We showed how commitment is superior to discretion. In particular, we learned that discretion leads to an inflationary bias.

We then talked about a model where the role of monetary policy does not arise because is physically costly (or impossible at times) to adjust prices, but because it was hard to acquire information about the state of the world (sticky information, and endogenous information acquisition), and we discussed about forward guidance. I'll make extra notes about forward guidance.

In this review session we will cover the main lessons of this semester for the flexible price economy and that with calvo prices, optimal monetary policy, and sticky information.

## 2 The Classical Dichotomy

This should be easy so I'll go fast over it. The easiest way to understand this concept is by considering a model like that of recitation 1. The representative agent can trade real and nominal bonds. We solve the problem and see that the nominal variables only matter to determine other nominal variables, the real variables are determined in another sub-block of the model. We call this result the classical dichotomy. Because nominal variables do not change real allocations, and the utility of the household only depends on real variables, then there is not a notion of the adequacy of different monetary policy schemes. Monetary policy does not matter in that model.

The lagrangean of this problem is:

$$\mathcal{L} = \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( \log C_t + \lambda_t (P_t Y_t + B_t(1+i_{t-1}) + P_t K_t (1+r_{t-1}) - P_t C_t - B_{t+1} - P_t K_{t+1} \right) \right]$$

The first order conditions are:

$$[C_t] : \frac{1}{C_t} = P_t \lambda_t \tag{1}$$

$$[B_{t+1}] : 1 = \mathbb{E}_t \left[ \beta(1+i_t) \frac{C_t}{C_{t+1}} \frac{P_t}{P_{t+1}} \right] \tag{2}$$

$$[K_{t+1}] : 1 = \mathbb{E}_t \left[ \beta(1+r_t) \frac{C_t}{C_{t+1}} \right] \tag{3}$$

And market clearing conditions state that bonds (both nominal and real) are in zero-net supply, and that the good-market clears.

$$C_t = Y_t \tag{4}$$

$$B_t = 0 \tag{5}$$

$$K_t = 0 \tag{6}$$

The euler equation, and the market clearing conditions $C_t = Y_t$ and $K_t = 0$ are sufficient for having an equilibrium. That is for a sequence of endowment shocks $Y$, consumption is given by $C_t = Y_t$, the interest rate adjusts to satisfy the Euler equation, and the real bond is in zero net supply. The nominal variables only enter in the nominal bond euler equation. So for a given sequence of expected inflation rates (pick one!) the interest rate is given by the Euler equation. Note there is not an equation that pins down the price level, any price level works, because agents do not suffer from money illusion.

## 3 The Flexible Price Equilibrium with Monopolistic Competition

We are going to introduce our standard model now. The new thing is that firms sell differentiated varieties. Demand is characterized by the elasticity of substitution $\sigma$ that governs how much households are willing to substitute across varieties, and intuitively, it should map into the slope of the demand curve. As in undergrad micro, the slope of the demand curve determines how much profits firms can earn, the more inelastic demand is, the less willing households are to substitute and therefore firms charge higher prices. Also, as in undergrad micro, higher prices and profits come at the cost of underproduction compared with perfect competition. In the aggregate we will show this translates into lower output.

### 3.1 Household

Let's start with the households

$$\max_{C_t, N_t} \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\psi}}{1+\psi}$$

$$C_t = \left( \int_0^1 C_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}$$

$$\int_0^1 P_{it} C_{it} di + B_{t+1} \leq W_t N_t + Profits_t + B_t(1 + i_{t-1})$$

The allocation of a given expenditure level across varieties is a static problem. Therefore we can divide the problem in two. One is a minimization expenditure problem, that is we decide what is the cheapest way to allocate a given consumption $C$ across all the $C_{it}$'s. This will give us a demand for each variety, and with that in the background, we can solve the intertemporal household problem only in terms of $C_t$.

$$\min_{C_{it}} \int_0^1 P_{it} C_{it} di$$

subject to

$$C_t = \left[ \int_0^1 C_{it}^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}$$

This problem is static. The Lagrangean is:

$$\mathcal{L} = \int_0^1 P_{it} C_{it} di + \lambda_t \left\{ C_t - \left[ \int_0^1 C_{it}^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}} \right\}$$

With FOC:

$$P_{it} = \lambda_t C_{it}^{-1/\sigma} \left[ \int_0^1 C_{it}^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{1}{\sigma-1}}$$

$$P_{it} = \lambda_t C_{it}^{-1/\sigma} C_t^{1/\sigma}$$

If we take the last equation elevated to the power $1-\sigma$, integrate over the varieties, and then take to the power $1/(1-\sigma)$ we find that:

$$
\begin{aligned}
P_{it} &= \lambda_t C_{it}^{-1/\sigma} C_t^{1/\sigma} \\
P_{it}^{1-\sigma} &= \left(\lambda_t C_{it}^{-1/\sigma} C_t^{1/\sigma}\right)^{1-\sigma} \\
\int_0^1 P_{it}^{1-\sigma} di &= \int_0^1 \left(\lambda_t C_{it}^{-1/\sigma} C_t^{1/\sigma}\right)^{1-\sigma} di \\
\left(\int_0^1 P_{it}^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} &= \left[\int_0^1 \left(\lambda_t C_{it}^{-1/\sigma} C_t^{1/\sigma}\right)^{1-\sigma} di\right]^{\frac{1}{1-\sigma}} \\
P_t &= \lambda_t C_t^{-1/\sigma} C_t^{1/\sigma} \\
P_t &= \lambda_t
\end{aligned}
$$

That is, the multiplier is equal to the price index. Which if we plug in the first order condition, and organize the terms yields:

$$
C_{it} = C_t \left(\frac{P_{it}}{P_t}\right)^{-\sigma} \tag{7}
$$

Which is the demand curve for variety $i$, which as expected has a constant elasticity of substituion equal to $\sigma$.

The lagrangean for the intertemporal problem is very standard. The solution is characterized by the following first order conditions. The first one, a labor supply equation, and the second, a consumption equation.

$$
N_t^{\psi} C_t^{\sigma} = \frac{W_t}{P_t} \tag{8}
$$

$$
C_t^{-\sigma} = \beta \mathbb{E}_t \left[C_{t+1}^{-\sigma} \frac{P_t}{P_{t+1}} (1 + i_t)\right] \tag{9}
$$

## 3.2 Firms

We are going to make firms as simple as possible.

$$
\max_{P_{it}} Profits_{it} = P_{it} Y_{it} - W_t N_{it}
$$

$$
\text{subject to}
$$

$$
Y_{it} = A_t N_{it}
$$

$$
C_{it} = Y_{it}
$$

$$
C_{it} = C_t \left(\frac{P_{it}}{P_t}\right)^{-\sigma}
$$

This maximization problem results in:

$$
P_{it} = \frac{\sigma}{\sigma - 1} \frac{W_t}{A_t}
$$

How come there is an optimal price if the production function is lineal? Because the profit function has curvature.

### 3.3 Equilibrium

Note that the RHS of the last equation does not depend on $i$. Therefore:

$$P_{it} = P_t$$

Which by the demand equation implies that

$$C_{it} = C_t$$

And by market clearing, and the production function $N_{it} = N_t$

Take now the labor supply schedule, and apply market clearing conditions, and the optimal price

$$\left(\frac{Y_t}{A_t}\right)^{\psi} Y_t^{\sigma} = \frac{W_t}{\frac{\sigma}{\sigma-1}\frac{W_t}{A_t}}$$

$$Y_t = \left(\frac{\sigma-1}{\sigma} A_t^{1+\psi}\right)^{\frac{1}{\sigma+\psi}}$$

In the economy without monopolistic competition, prices are equal to marginal costs:

$$P_t^{pc} = \frac{W_t^{pc}}{A_t}$$

$$Y_t^{pc} = \left(A_t^{1+\psi}\right)^{\frac{1}{\sigma+\psi}}$$

You can see that the inefficiency in the micro level adds-up to inefficiencies in aggregate output. This is not surprising, all the firms are identical. We can fix this inefficiency by a subsidy on the payroll of firms, you did this in a problem set, so I won't do it again.

# 4 Price Rigidity

### 4.1 Calvo Model

We found how target prices look in the Calvo model, and you should be able to derive them:

$$P_t^* = \frac{\sigma}{\sigma-1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k C_{t+k}^{1-\gamma} P_{t+k}^{\sigma} MC_{t+k|k}}{\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k C_{t+k}^{1-\gamma} P_{t+k}^{\sigma-1}} \tag{10}$$

Which in its log-linearized form is:

$$p_t^* = (1-\beta\theta)\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k \left[\hat{mc}_{t+k|k} + p_{t+k}\right]$$

$$p_t^* - p_{t-1} = -p_{t-1} + (1-\beta\theta)\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k \left[\hat{mc}_{t+k|k} + p_{t+k}\right]$$

Now take the first term from the sum:

$$p_t^* - p_{t-1} = -p_{t-1} + (1-\beta\theta)\hat{mc}_{t|0} + p_t - \beta\theta p_t + (1-\beta\theta)\beta\theta\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k \left[\hat{mc}_{t+k+1|k} + p_{t+k+1}\right]$$

$$p_t^* - p_{t-1} = -p_{t-1} + (1-\beta\theta)\hat{mc}_{t|0} + p_t - \beta\theta p_t + \beta\theta\mathbb{E}_t p_{t+1}^*$$

$$p_t^* - p_{t-1} = p_t - p_{t-1} + (1-\beta\theta)\hat{mc}_{t|0} + p_t - \beta\theta p_t + \beta\theta\mathbb{E}_t(p_{t+1}^* - p_t)$$

Now, in loglinear terms:

$$p_t = \theta p_{t-1} + (1-\theta)p_t^*$$

Which implies:

$$\pi_t = (1-\theta)(p_t^* - p_{t-1})$$

If we apply this last relationship we get:

$$\frac{\pi_t}{1-\theta} = \pi_t + (1-\beta\theta)\hat{mc}_t + \frac{\beta\theta}{1-\theta}\mathbb{E}_t\pi_{t+1}$$

Which in turns implies the NKPC:

$$\pi_t = \frac{(1-\beta\theta)(1-\theta)}{\theta}\hat{mc}_t + \beta\mathbb{E}_t\pi_{t+1}$$

This is already the NKPC. You can express it in terms of the output gap given the relationship of this object with respect to marginal costs. Note that if we change the production function to make it have decreasing returns to scale, there would be strategic complementarities, and this would be reflected in an additional coefficient in front of the marginal cost coefficient, making the Phillips curve flatter.

The system is then characterized by three equations in log-linear form:

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa\tilde{y}_t$$
$$\tilde{y}_t = \beta\mathbb{E}_t\tilde{y}_{t+1} - \frac{1}{\gamma}(i_t - \mathbb{E}_t\pi_{t+1} - r_t^n)$$
$$i_t = \phi_\pi\pi_t + \phi_y\tilde{y}_t + u_t$$

What do we know about the Taylor Rule? We know the coefficients of the equation need to satisfy certain restrictions for the economy to not blow up. Will do the simple case:

How does a

## 4.2 Taylor Principle

Please note that what I will do now has nothing to do with any assumptions about price rigidity, it is true even if prices are flexible.

Consider the interest rate rule:

$$i_t = \phi(\pi_t - \bar{\pi})$$

And the Fisher rule

$$i_t = r_t + \mathbb{E}_t\pi_{t+1}$$

Plug the monetary policy rule in the fisher rule:

$$\pi_t = \frac{1}{\phi}\left(r_t + \mathbb{E}_t\pi_{t+1} + \phi\bar{\pi}\right)$$

Iterate forward:

$$\pi_t = \sum_{k=0}^{\infty}\phi^{-(k+1)}\left(\mathbb{E}_t r_{t+k} + \phi\bar{\pi}\right)$$

If we want to $\pi_t$ to be bounded, we need $\phi > 1$. We call this the Taylor principle. In more complicated rules it is difficult to do the replace-iterate-forward procedure. We use the Blanchard-Kahn conditions for those cases.

# 5 Optimal Policy

I will take as given the loss function, not enough time in recitation to derive it. To form your intuition look at the problem of the monetary authority.

$$\min \mathbb{E}_t \sum_{t=0}^{\infty} (\pi_t^2 + \lambda \tilde{y}_t)$$

Subject to:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \tilde{y}_t$$

Eyeball optimization: zero output gap and zero inflation minimize the objective. They also satisfy the constraint. Not much to argue, zero inflation and zero output gap are optimal. How to implement?

$$\tilde{y}_t = \beta \mathbb{E}_t \tilde{y}_{t+1} - \frac{1}{\gamma}(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n)$$

For zero inflation and zero output gap, the euler equation imply that the interest rate mirrors the natural rate. This result, that zero output gap and zero inflation are compatible is called the divine coincidence.

## 5.1 Commitment vs. Discretion

Let's first look what happens if there is discretion. Perhaps central bankers change every period, so the current central banker cannot control $\pi_{t+1}$ and therefore takes epectations as given.

The loss function is given by:

$$L_t = \pi_t^2 + \lambda(\tilde{y}_t - \tilde{y}^*)$$

Where $y^*$ is the optimal output gap.

The central bank wants to minimize subject to the phillips curve

$$\pi_t = \beta \pi^e + \kappa \tilde{y}_t$$

We take first order conditions and find

$$\hat{y}_t = -\frac{\kappa \pi_t}{\lambda}$$

We can replace this in the Phillips curve and find:

$$\pi_t = \left(1 + \frac{\kappa^2}{\lambda}\right)^{-1} (\kappa \tilde{y}^* + \beta \pi^e)$$

Because we have rational expectations, it has to be the case that the expectations coincide with the inflation, so:

$$\pi^e = \frac{\kappa \lambda}{(1-\beta)\lambda + \kappa^2} \tilde{y}^* > 0$$

If we were to solve the commitment problem, we would see that inflation goes to zero asymptotically even if $\hat{y}^* > 0$. This means that the discretion problem has an inflationary bias.

# 6 Sticky Information

I will use the notation on the published version of the paper. If you prefer another one let me know and I can update it. A fraction $\lambda$ of producers adjust their information set every period. This is also a fairy, but not one that lets you to adjust your price (you are free to do that), but lets you to adjust your information set (to open your eyes and look at the world).

The average price in the economy.

$$p_t = \lambda \sum_{t=0}^{\infty} (1-\lambda)^j x_t^j$$

Where $x_t^j$ is the (log) price set by firms who adjusted their information $j$ periods ago.

Firms set prices equal to the expectations they had the last time they adjusted their information set about optimal prices today:

$$x_t^j = \mathbb{E}_{t-j} p_t^*$$

And this optimal price is given by:

$$p_t^* = p_t + \alpha y_t$$

Which we can write also (for fun) as:

$$p_t^* = (1-\alpha) p_t + \alpha m_t$$

That is, there are strategic complementarities in the model. We can derive that desired price, but we will not do it here.

Putting those three equations together we get:

$$p_t = \lambda \sum_{t=0}^{\infty} (1-\lambda)^j \mathbb{E}_{t-j}(p_t + \alpha y_t)$$

You can see that the current price level is a combination of *past expectations of current* output and prices, instead of being a current *current expectations of future* variables as in the Calvo model.

Start by taking out the first term in the sumation:

$$p_t = \lambda(p_t + \alpha y_t) + \lambda \sum_{t=0}^{\infty} (1-\lambda)^{j+1} \mathbb{E}_{t-j-1}(p_t + \alpha y_t)$$

Rearrange:

$$p_t(1-\lambda) = \lambda \alpha y_t + \lambda(1-\lambda) \sum_{t=0}^{\infty} (1-\lambda)^j \mathbb{E}_{t-j-1}(p_t + \alpha y_t)$$

$$p_t = \frac{\lambda \alpha}{1-\lambda} y_t + \lambda \sum_{t=0}^{\infty} (1-\lambda)^j \mathbb{E}_{t-j-1}(p_t + \alpha y_t)$$

Define also the price yesterday:

$$p_{t-1} = \lambda \sum_{t=0}^{\infty} (1-\lambda)^j \mathbb{E}_{t-j-1}(p_{t-1} + \alpha y_{t-1})$$

Substract the price yesterday from the price today:

$$\pi_t = \frac{\lambda \alpha}{1-\lambda} y_t + \lambda \sum_{t=0}^{\infty} (1-\lambda)^j \mathbb{E}_{t-j-1}(\pi_t + \alpha \Delta y_t)$$

Which is the Sticky Information Phillips Curve, or the Mankiw-Reis Phillips Curve.

# 7 Calvo vs. Sticky Information

Take the Calvo Phillips Curve:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa y_t$$

Evaluate it in steady state:

$$\pi = \beta \pi + \kappa y$$

Solve for $y$ as a function of $\pi$

$$\pi = \frac{\kappa}{1-\beta} y$$

It violates the natural rate property, that is we want a Phillips Curve that in steady state gives a 0 output gap.

Now iterate forward:

$$\pi_t = \kappa \sum_{j=0}^{\infty} \beta^k \mathbb{E}_t y_{t+k}$$

For whatever monetary policy rate you should be convinced by now that the real effects of a shock today in the output gap in the distant future tends to zero, and that it is positive in the trajectory. Therefore inflation jumps and then goes down, at odds with the data that predicts hump-shaped responses of inflation.

Sticky information solves these problems. It produces hump shapes in inflation, and the natural rate hypothesis works fine.

What's the catch? We saw in recitation how taking the Calvo NK Phillips Curve to the data is already challenging, the Sticky information Phillips Curve is even more challenging. Need to have measures of past expectations of current inflation and output growth. Incredibly hard to come up with them.