The Quantity Theory of Money

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1 Goal

Inflation is the increase in the *overall* price level. You should think of it as the decrease in the purchasing power of money. We see in the data that when inflation gets out of control, the population of the countries suffer, and economic conditions deteriorate. It is difficult to say whether the change in inflation *causes* the decline of economic conditions. To determine this causality we have to options.

1. Experiment. We could change a determinant of inflation at random, and then we could check what happens to the countries or regions that had higher inflation than others.
2. We can postulate theories about what determines inflation. How is it tied with economic conditions. Check the implications of those theories against the data, and try to learn from there.

To do any of those two options, we need to understand where inflation comes from. Today we analyze a family of theories, broadly called under the umbrella of the **Quantity Theory of Money**. We rarely do 1). Experimenting with inflation seems to be very irresponsible, we believe that policymakers are really careful in implementing policies that affect the overall level of prices. That important focus on how to conduct monetary policy, makes answering the questions related to the determinants and consequences of inflation really hard. Come talk to me if you want to discuss further.

2 The Quantity Theory of Money

Money has three main functions. Serve as unit of account, means of exchange, and store of value. The QTM is grounded on the role of money as a means to exchange. People hold money to be able to buy stuff with it, therefore it is natural to relate the *stock* of money to the nominal value *flow* of production (called GDP!)

$$M_t^d = \frac{1}{v_t} P_t Y_t$$

Money is a stock, but GDP is a flow. a good rule of thumb is that you cannot equate stocks to flows, you need to adjust the stock to make it comparable with the flow, and that is what the number v is doing (we will define it now).

Imagine a world in which in at the beginning of the year first we get ten dollars, and we make exactly one purchase over the year. The transactions of this economy are ones in which we produce and sell a widget to the person next to us. Therefore the value of the transactions that happened in this fyctional economy has to be equal to the sum of the 10 dollars each of us has. That is, $M = PY$. We all finish the year with 10 dollars in hand, since we made purchases of 10 bucks and made sales for 10 bucks.

Now imagine a second economy, in which we receive the same 10 dollars, but we make two purchases, from the two people next to us. We also end the year with 10 bucks at hand, but this time nominal GDP (the value of goods produced) is double (every one of us made two purchases of 10 dollars). In that case the number of dollars in this economy is half of nominal GDP.

The equation is saying that if on average we make v purchases, then the stock of money is $1/v$ the value of GDP.

We are going to start making a very simple but fundamental assumption, that v is constant.

Dividing $M_t^d = \frac{1}{v} P_t Y_t$ over $M_{t-1}^d = \frac{1}{v} P_{t-1} Y_{t-1}$ we get:

$$1 + \mu_t = (1 + \pi_t)(1 + g_t)$$

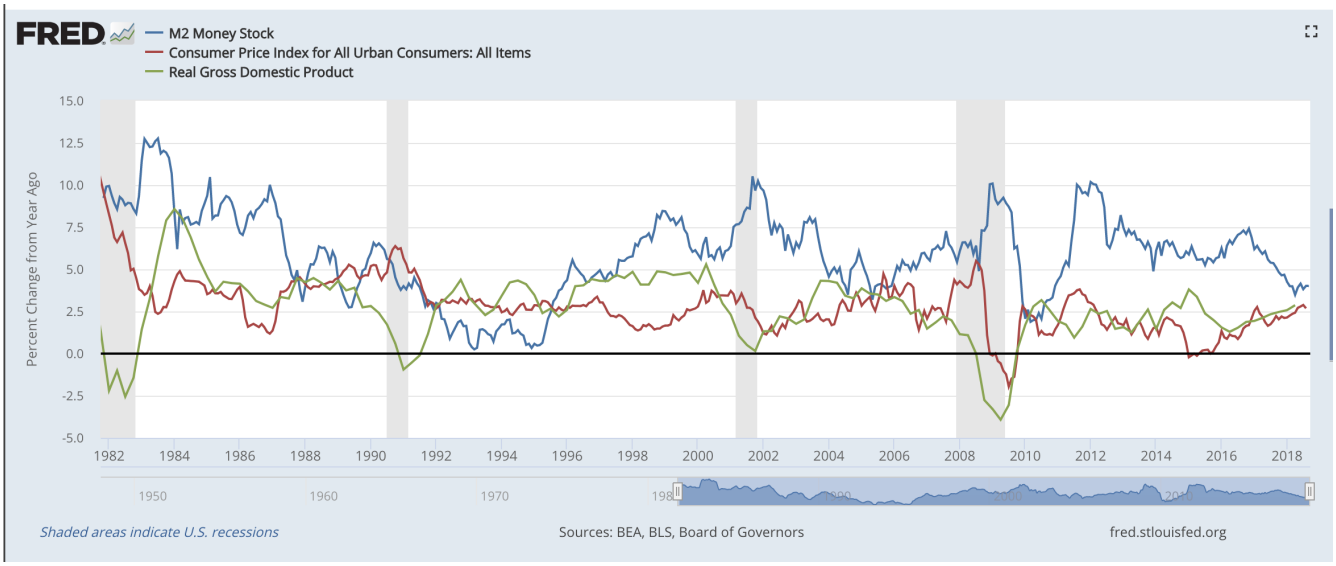
. When μ, π, and g are small, then it is approximately true that:

$$\mu_t \approx \pi_t + g_t$$

That is, when the stock of money grows it has to be the case that inflation increased or that the economy grew. If we had not assumed that velocity is constant, there would be another term, summarizing how the velocity changed!

The theory is completed by another step, that the central bank controls the supply of money, and therefore it controls μ. So changes in μ have to change inflation or change the rate at which output grows.

This theory is difficult to test. There are three parts moving, but most importantly, we believe the Federal Reserve or other central banks are smart. So if they expect that in the future the either π or g is changing, they have motives to change μ to alter that path. So there are a lot of *reverse causality* concerns.



I want to highlight here that we should expect the blue line to move similarly to the green and red lines, but in the short run exactly the opposite happens. It is true that in the 80's, when μ was higher, also inflation was higher, but while inflation fell in the recession μ increased!

This could be the reverse causality we talked about. Knowing that inflation was going down, the federal reserve increased μ to make inflation to fall less, we end up seeing a negative correlation but the theory could still be right. Macroeconomists have put together a lot of empirical and theoretical work into answering this question correctly.

Our model was just too simple. let's add a little bit of more detail, specially to understand what is v, the velocity of money.

3 The Baumol Tobin Model

A person earns PY and receives her income at the beginning of the period, and spends her money at a constant rate. The money is deposited in her bank account. The good thing about having money in the savings account is receiving interest income. The bad thing is that there are costs, of taking your money out, this could be because they literally charge you, or because you have to spend time or money going to the ATM.

That can be represented in the following equation:

$$\text{Cost} = i \times \textbf{Money held in Cash} + n \times PC$$

The idea is that when the interest rate is higher, you will have less money in cash, and when the cost is higher you will hold more, since you can save on trips.

We arrive at the notion that the money you held in hand is equal to $PY/(2n)$. But where does that come from? The answer is geometry. When you go only once to the bank you retrieve PY dollars, and then you consume them continuously. That looks like the triangle in the lecture notes. The area of that triangle is $(PY \times 1)/2$. If you go twice, you will have two triangles. Each of them with base $1/2$ (half of the period), height $PY/2$, so the area of each triangle is

$(1/2 \times PY/2)/2 = PY/8$. But you have two of those, that is the total money is $2 \times PY/8 = PY/4$. If you keep doing that you notice a pattern, that the total money in hand is equal to $PY/(2n)$.

It is easy to show that the money held in cash is the total amount of income PY divided by 2 times the number of trips.

Therefore:

$$\text{Cost} = i \times \frac{PY}{2n} + n \times PC$$

. So when n increases it raises one cost, and diminishes the other. How many trips will people make? The number that minimizes the cost:

$$\frac{\partial \text{Cost}}{\partial n} = 0$$

Taking that derivative we find that:

$$n^* = \sqrt{\frac{iY}{2C}}$$

. That is, when the reward for keeping money in the bank is higher people make more trips. When is more costly to go people make less trips. And when there is more money to spend, then people make more trips.

Therefore the money held in cash is equal to $\frac{PY}{2n^*}$, or $M^d = P\sqrt{\frac{YC}{2i}}$. If we further impose that the cost is proportional to the income Y, then:

$$M_t^d = \sqrt{\frac{c}{2i}} PY$$

Or,

$$M_t^d = \frac{1}{v_t} PY$$

, the QTM equation, but knowing what v depends on the interest rate. So if the interest rate changes the velocity will change.